Section 4.1 Angles and Their Measures

EXPLORATION 1 **Constructing a 1-Radian Angle**

Carefully draw a large circle on a piece of paper, either by tracing around a circular object or by using a compass. Identify the center of the circle (O) and draw a radius horizontally from O toward the right, intersecting the circle at point A. Then cut a piece of thread or string the same size as the radius. Place one end of the string at A and bend it around the circle counterclockwise, marking the point B on the circle where the other end of the string ends up. Draw the radius from O to B.

The measure of angle AOB is 1 rad.

1. What is the circumference of the circle, in terms of its radius r
2. How many radians must there be in a complete circle?
3. If we cut a piece of thread 3 times as big as the radius, would it extend halfway around the circle? Why or why not?
4. How many radians are in a straight angle?

Trigonometry means ________________ _____ ________________.

In the space below <u>draw a picture of an angle</u> and <u>label</u> the **vertex**, **initial side** and the **terminal side**.

An ________________ is determined by rotating a ray about its endpoint. The starting position of the ray is the ________________ ___________ of the angle, and the position after rotation is the ________________ ___________. The vertex is the ________________ of the ray. An angle that fits the coordinate system in which the origin is the vertex and the initial side coincides with the positive x-axis is an angle in ________________ ________________. Counterclockwise rotation generates ________________ ___________ while clockwise rotation generates ________________ ___________.

How are angles labeled?

Angles that have the same initial and terminal sides are called ________________ angles. In the space below draw an example of coterminal angles.

A measure of an angle is determined by the ________________ _____ ________________ from the initial side to the terminal side.

One way to measure angles is in ______________. Another way to measure angles is in ______________.

One **radian** is the measure of a ________________ ___________ _____ that intercepts an arc _____ equal in length to the radius _____ of the circle.

In other words, $\theta = \dfrac{s}{r}$ where θ is measured in radians. (Note that $\theta = 1$ when $s = r$.)

Remember, the circumference of a circle is $2\pi r$ units and it follows that a central angle of one full revolution (counterclockwise) corresponds to an arc length of $s = 2\pi r$. Also recall that there are approximately ___________ ___________ ___________ in a full circle ($2\pi \approx 6.28$).

Conversions Between Degrees and Radians

1. To convert degrees to radians, multiply degrees by $\dfrac{\pi\ rad}{180^\circ}$.
2. To convert radians to degrees, multiply radians by $\dfrac{180^\circ}{\pi\ rad}$.

Example 1: Express each of the following angles in radian measure as a multiple of π.

(Do not use a calculator.)

a. 420° b. 280° c. – 30°

Example 2: Express the following angles in degree measure. (Do not use a calculator.)

a. $\dfrac{\pi}{9}$ b. $\dfrac{8\pi}{3}$ c. 3 radians

Arc Length

For a circle of radius *r*, a central angle θ intercepts an arc length *s* given by $s = r\theta$ where θ is measured in radians.

Note that if $r = 1$, then $s = \theta$, and the radian measure of θ equals the arc length.

Example 3:

A circle has a radius of 10 inches. Find the length of the arc intercepted by a central angle of 140°.

Linear and Angular Speeds

Consider a particle moving at a constant speed along a circular arc of radius *r*. If *s* is the length of the arc traveled in time *t*, then the **linear speed** *v* of the particle is:

$$\text{Linear speed } v = \frac{arc\ length}{time} = \frac{s}{t}.$$

Moreover, if θ is the angle (in radian measure) corresponding to the arc length *s*, then the **angular speed** ω (the lowercase Greek letter omega) of the particle is:

$$\text{Angular speed } \omega = \frac{central\ angle}{time} = \frac{\theta}{t}.$$

Example 4:

The second hand of a watch is 1.3 centimeters long. Find the linear speed of the tip of this second hand as it passes around the watch face.

Example 5:

The circular blade on a saw rotates at 4200 revolutions per minute.

a. Find the angular speed in radians per second.

b. The blade has a radius of 6 inches. Find the linear speed of a blade tip in inches per second.

A ________________ of a circle is a region bounded by two radii of the circle and their intercepted arc.

Area of a Sector of a Circle
For a circle of radius r, the area A of a sector of the circle with central angle θ is:
$A = \frac{1}{2} r^2 \theta$ where θ is measured in radians.

Example 6:

A sprinkler on a golf course is set to spray water over a distance of 75 feet and rotates through an angle of 135°. Find the area of the fairway watered by the sprinkler.